Coupled-channel evaluations of cross sections for scattering involving particle-unstable resonances

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How does the scattering cross section change when the colliding bound-state fragments are allowed particle-emitting resonances? This question is explored in the framework of a multi-channel algebraic scattering method of determining nucleon-nucleus cross sections at low energies. Two cases are examined, the first being a *gedanken* investigation in which $n+^{12}$C scattering is studied with the target states assigned artificial widths. The second is a study of neutron scattering from ^{8}Be; a nucleus that is particle unstable. Resonance character of the target states markedly varies evaluated cross sections from those obtained assuming stability in the target spectrum.

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The ready availability of radioactive ion beams allows experimental information to be obtained on many exotic nuclei, allowing for study of novel structures, such as skins and halos. Of particular interest are the data obtained from scattering exotic nuclei from hydrogen targets, which equates to proton scattering from those nuclei in the inverse kinematics. Such data have been analysed in terms of effective nucleon-nucleus interactions used in distorted wave approximations [1, 2, 3], or using coupled-channels approaches [4, 5].

This paper considers the situation of low-energy neutron scattering from two light mass nuclei (^{12}C and ^{8}Be) for which discrete resonance effects in the elastic cross section are usually present. Such resonance properties most often result from channel coupling and are reproduced here using a multi-channel algebraic scattering (MCAS) theory [5]. With MCAS, solutions of coupled Lippmann-Schwinger equations are found (in momentum space) by using finite-rank separable representations of an input matrix of nucleon-nucleus interactions. An "optimal" set of sturmian functions [6] is used as the expansion set. Details are given in Refs. [5, 7]. The advantages of using the MCAS method include an ability to locate all compound system resonance centroids and widths regardless of how narrow those resonances may be, and, by use of orthogonalizing pseudo-potentials (OPP) in generating sturmians, to ensure the Pauli principle is not violated [7], despite the collective model formulation of nucleon-nucleus interactions used therein. The latter is of paramount importance for coupled-channel calculations [8], as otherwise some compound nucleus states so defined possess spurious components in their wave functions.

MCAS is used to find solutions of the coupled-channel, partial-wave expanded Lippmann-Schwinger equations for each total system spin-parity (J^π),

$$T^{J^\pi}_{cc'}(p,q;E) = V^{J^\pi}_{cc'}(p,q) + \mu\left[\sum_{c''=1}^{\text{open}} \int_0^\infty V^{J^\pi}_{cc''}(p,x)\frac{x^2}{k^2_{c''}-x^2+i\varepsilon}T^{J^\pi}_{c''c'}(x,q;E)\,dx - \sum_{c''=1}^{\text{closed}} \int_0^\infty V^{J^\pi}_{cc''}(p,x)\frac{x^2}{h^2_{c''}+x^2}T^{J^\pi}_{c''c'}(x,q;E)\,dx\right], \quad (1)$$

where a finite set of scattering channels, denoted c, are considered, and where $\mu = \frac{2\overline{m}}{\hbar}$, $\overline{m}$ being the reduced mass. There are two summations as the open and closed channel components are separated, with wave numbers

$$k_c = \sqrt{\mu(E-\epsilon_c)} \text{ and } h_c = \sqrt{\mu(\epsilon_c - E)}\,, \quad (2)$$

for $E > \epsilon_c$ and $E < \epsilon_c$ respectively. ϵ_c is the energy threshold at which channel c opens (the excitation energies of the target nucleus). Henceforth the J^π superscript is to be understood. Expansion of $V_{cc'}$ in terms of a finite number (N) of sturmians leads to a separable represen-

tation of the scattering matrix [5]

$$S_{cc'} = \delta_{cc'} - i^{(l_{c'} - l_c + 1)} \pi \mu \times \sum_{n,n'=1}^{N} \sqrt{k_c} \hat{\chi}_{cn}(k_c) \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \hat{\chi}_{c'n'} \sqrt{k_{c'}} \,, \quad (3)$$

where c and c' refer now only to open channels, l_c is the partial wave with channel c and the Green's function matrix is

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2} \hat{\chi}_{cn'}(x)\, dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x)\, dx \right]. \quad (4)$$

$\boldsymbol{\eta}$ is a column vector of sturmian eigenvalues and $\hat{\chi}$ are form factors determined from the chosen sturmian functions. Details are given in Ref [5].

Traditionally, all target states are taken to have eigenvalues of zero width and the (complex) Green's functions are evaluated using the method of principal parts. This assumes time evolution of target states is given by

$$|x, t\rangle = e^{-iH_0 t/\hbar} \; |x, t_0\rangle = e^{-iE_0 t/\hbar} \; |x, t_0\rangle \,. \quad (5)$$

However, if states decay, they evolve as [9]

$$|x, t\rangle = e^{-\frac{\Gamma}{2} t} \; e^{-iE_0 t/\hbar} \; |x, t_0\rangle \,. \quad (6)$$

Thus, in the Green's function, channel energies become complex, as do the squared channel wave numbers,

$$\hat{k_c}^{\,2} = \mu \left(E - \epsilon_c + \tfrac{i\Gamma_c}{2} \right) \; ; \; \hat{h_c}^{\,2} = \mu \left(\epsilon_c - E - \tfrac{i\Gamma_c}{2} \right) , \quad (7)$$

where $\frac{\Gamma_c}{2}$ is half the width of the target state associated with channel c. Thus, the Green's function matrix elements are

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + \frac{i\mu\Gamma_c}{2}} \hat{\chi}_{cn'}(x)\, dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2 - \frac{i\mu\Gamma_c}{2}} \hat{\chi}_{cn'}(x)\, dx \right] , \quad (8)$$

where k_c and h_c are as in Eq. (2). Thus, poles are moved significantly off the real axis, and integration of a complex integrand along the real momentum axis is feasible. This has been done; however, for any infinitesimal-width target state, or resonance so narrow that it can be treated as such, the method of principal parts has been retained.

As previously [5, 7], the ^{13}C (n+^{12}C) system is studied using the MCAS approach with a rotational model prescription of the matrix of interaction potentials connecting three states of ^{12}C (the $0^+_{g.s.}$, 2^+_1 (4.43 MeV) and 0^+_2 (7.64 MeV)), using the same interaction Hamiltonian and allowing for Pauli blocking via the OPP scheme. In the first instance, all three states are considered zero-width, giving the elastic scattering cross section of neutrons to 6 MeV as previously published. Additionally, evaluations are made for the same interaction allowing the 2^+_1 and 0^+_2 states of ^{12}C to have particle emission widths of varying size; the ground state kept with zero width. Results are displayed in Fig. 1.

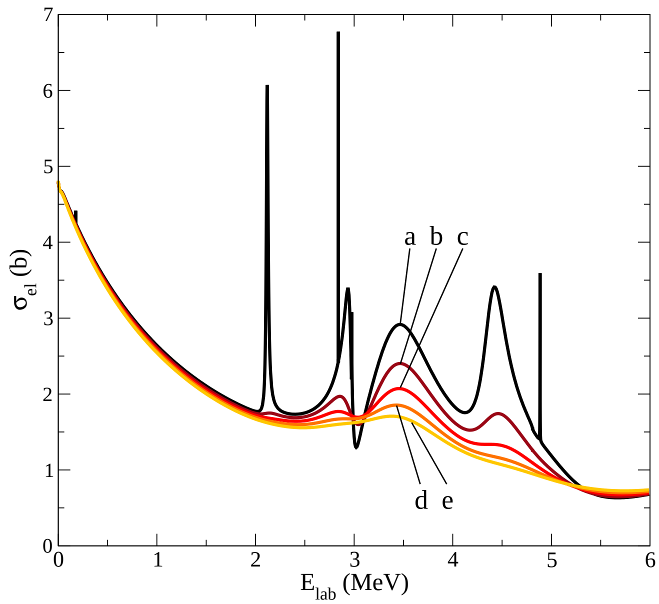

FIG. 1: (Color online) Calculated cross sections for hypothetically n-(unstable) ^{12}C scattering as functions of neutron energy. Labels as per Table I.

Cross sections follow a marked trend as hypothetical state widths increase. Widths used are listed in Table I. Ascribing the first widths to the excited states (set (b)), very narrow resonances in the original cross section disappear. From the earlier studies [5, 7], it is noted that those (narrow) compound resonance states are dominated by the coupling of an sd-shell nucleon to the 2^+_1 state in ^{12}C. The broader resonances remain evident in the cross section as the state widths are artificially increased. However, with these increases the remaining resonances smear out. In the case of the broadest target states (set (e)) the

TABLE I: Artificial widths (Γ, in MeV) assigned to ^{12}C eigenstates.

Curve	0^+_1 width	2^+_1 width	0^+_2 width
a	—	—	—
b	0.00	0.20	0.60
c	0.00	0.40	1.20
d	0.00	0.60	1.80
e	0.00	0.80	2.40

cross section has very little remnant of the compound system resonances. Clearly only the cross section from evaluation with three zero-width target states replicates measurement.

Table II displays the widths of states in the compound nucleus, ^{13}C, found using MCAS when attributing the diverse widths to the excited states of ^{12}C listed in Table I. The first column after J^π lists the bound state and resonance centroid energies obtained from the calculation made with the physically reasonable, zero-width excitation energies of the 2_1^+ and 0_2^+ states of ^{12}C. Allowing those states to be resonances with the widths selected alters the state energy centroids by at most a few tens of keV, so these are not listed. Thus, it is observed that allowing these target states to be resonances mostly affects widths of the resulting compound nucleus resonances. Those variations are consistent with changes noted in the cross section, with sharp resonances found for the zero-width state case rapidly disappearing and the others broadening to an extent that only a few are left distinguishable from a background. It is important to note, though, that all states in the compound system defined by the coupled-channel evaluations remain present, with, in this case, centroid energies little affected but widths increased.

TABLE II: Widths (in MeV) for ^{13}C states from calculation allowing states of ^{12}C to be resonances as listed in Table I.

J^π	Centroid	(a)	(b)	(c)	(d)	(e)
$\frac{1}{2}^-$	−4.82	—	—	—	—	—
$\frac{1}{2}^+$	−2.04	—	—	—	—	—
$\frac{5}{2}^+$	−1.85	—	—	—	—	—
$\frac{3}{2}^-$	−1.36	—	—	—	—	—
$\frac{5}{2}^-$	0.16	7×10^{-10}	0.09	0.18	0.28	0.37
$\frac{5}{2}^+$	1.95	0.01	0.10	0.19	0.28	0.38
$\frac{7}{2}^+$	2.62	9×10^{-7}	0.09	0.18	0.28	0.37
$\frac{3}{2}^+$	2.74	0.04	0.13	0.23	0.33	0.42
$\frac{1}{2}^-$	2.75	8×10^{-4}	0.27	0.55	0.82	1.11
$\frac{3}{2}^+$	3.25	0.45	0.50	0.56	0.61	0.67
$\frac{5}{2}^+$	4.06	0.13	0.25	0.38	0.52	0.66
$\frac{9}{2}^+$	4.51	7×10^{-4}	0.09	0.19	0.28	0.38
$\frac{1}{2}^+$	4.76	0.52	0.71	0.92	1.14	1.37

The low excitation ^{8}Be spectrum has a 0^+ ground state that has a small width for its decay into two α-particles (6×10^{-6} MeV), a broad 2^+ resonance state with centroid at 3.03 MeV and width of 1.5 MeV, followed by a broader 4^+ resonance state with centroid at 11.35 MeV and width $\sim$3.5 MeV [10]. Two evaluations of the n+^{8}Be cross section are obtained with MCAS; in both, the ground state is taken as having zero-width. In the first, both the 2^+ and 4^+ states are also taken as zero-width (ignoring their known α-decay widths) whereas in the second, the empirical widths are used. In both calculations, the same nuclear interaction is considered. It is taken from a rotor model with parameter values chosen in the finite-width states calculation to reproduce some aspects of the experimentally determined structure of ^{9}Be [10], shown graphically in Fig. 2.

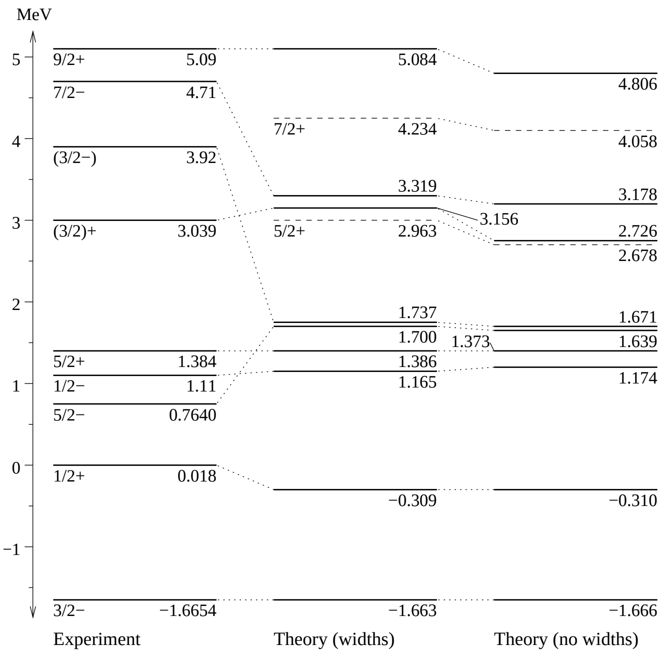

FIG. 2: Experimental ^{9}Be spectrum and that calculated from neutron scattering with stable and unstable ^{8}Be.

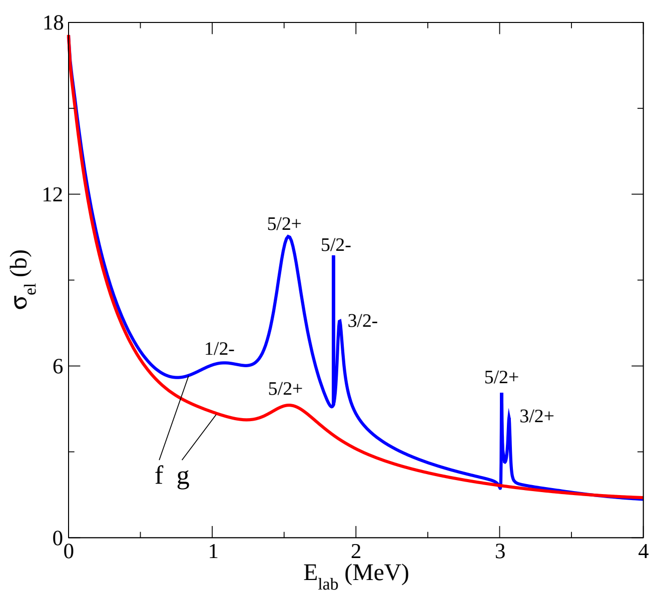

FIG. 3: (Color online) Calculated cross sections for neutron scattering from (f) stable and (g) unstable ^{8}Be as functions of neutron energy.

The results for the scattering cross sections are shown in Fig. 3. Upon introducing target state widths, as found in the $n+{}^{12}$C investigation, the resonances are suppressed but still present; their widths increasing and magnitudes decreasing so as all but the $\frac{5}{2}^+$ cannot be discerned from the background. Compound system resonances of both calculations are at essentially the same energies. These effects are further illustrated by the centroid energies and widths of the resonances listed in Table III. Column 1 (after J^π) lists the resultant centroid energies of the spectrum found assuming the 2_1^+ and 4_1^+ excited states in ^{8}Be have zero width, and the widths of these resonances (energies above the $n+{}^8$Be threshold) are in column 2. Columns 3 and 4 list energies and widths, respectively, obtained using the MCAS scheme on allowing the two excited states in ^{8}Be to have their ascribed widths. Column 5 lists the experimental widths [10]. As found in the study of the ^{13}C spectrum, taking the excited states of ^{8}Be to be resonances gives the same spectral list as when they are treated as zero-width, but the widths of the compound nuclear states found significantly increase. This is again reflected in the cross sections. These increases bring the theoretical ^{9}Be state widths closer, often significantly, to experimental values. In this case, some centroid energies are shifted by up to 330 keV.

TABLE III: ^{9}Be state centroids and widths (E & Γ in MeV) from calculation with ^{8}Be states taken as zero-width and then with known resonance widths, and experimental widths.

	Zero-width		Resonances		Experiment
J^π	E(f)	Γ(f)	E(g)	Γ(g)	Γ(exp.)
$\frac{3}{2}^-$	−1.67	—	−1.66	—	—
$\frac{1}{2}^+$	−0.31	—	−0.31	—	0.217±0.001
$\frac{1}{2}^-$	1.17	0.646	1.16	0.972	1.080±0.110
$\frac{5}{2}^+$	1.37	0.118	1.39	0.244	0.282±0.011
$\frac{5}{2}^-$	1.64	3.7×10^{-9}	1.70	0.694	7.8×10^{-4}
$\frac{3}{2}^-$	1.67	0.022	1.74	0.682	1.330±0.360
$\frac{5}{2}^+$	2.68	0.003	2.96	1.141	N/A
$\frac{3}{2}^+$	2.73	0.009	3.16	1.856	0.743±0.055
$\frac{7}{2}^-$	3.18	0.009	3.32	0.786	1.210±0.230
$\frac{7}{2}^+$	4.06	0.072	4.23	0.873	N/A
$\frac{9}{2}^+$	4.81	0.189	5.08	1.261	1.330±0.090

In conclusion, a multi-channel algebraic scattering approach to theoretical evaluation nucleon-nucleus scattering information has been extended to consider widths of target nucleus eigenstates. Resultant resonances in obtained cross sections suffer only minor changes to their energy centroids. However, their widths are substantially increased, often making them indistinguishable from the scattering background, but closer to experimentally determined widths. This was observed for $n+{}^8$Be scattering using calculations with and without experimental target state widths. Furthermore, it is found that this effect increases as target state widths increase, with sharp resonances quickly obscured. This was observed from calculations using a series of artificial target state widths.

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